1. Let f be a function such that $f(x+y) = f(x) + f(y)$ for all x and y. Assume $f(5) = 9$. Compute $f(2015)$.

2. There are six cards, with the numbers $2, 2, 4, 4, 6, 6$ on them. If you pick three cards at random, what is the probability that you can make a triangles whose side lengths are the chosen numbers?

3. A train travels from Berkeley to San Francisco under a tunnel of length 10 kilometers, and then returns to Berkeley using a bridge of length 7 kilometers. If the train travels at 30 km/hr underwater and 60 km/hr above water, what is the train's average speed in km/hr on the round trip?

4. Given a string consisting of the characters A, C, G, U, its reverse complement is the string obtained by first reversing the string and then replacing A's with U's, C's with G's, G's with C's, and U's with A's. For example, the reverse complement of $UAGCAC$ is $GUGCUA$. A string is a palindrome if it's the same as its reverse. A string is called self-conjugate if it's the same as its reverse complement. For example, $UAGGAU$ is a palindrome and $UAGCUA$ is self-conjugate. How many six letter strings with just the characters A, C, G (no U's) are either palindromes or self-conjugate?

5. A scooter as 2 wheels, a chair has 6 wheels, and a spaceship has 11 wheels. If there are 10 of these objects, with a total of 50 wheels, how many chairs are there?

6. How many proper subsets of $\{1, 2, 3, 4, 5, 6\}$ are there such that the sum of the elements in the subset equal twice a number in the subset?

7. A circle and square share the same center and area. The circle has radius 1 and intersects the square on one side at points A and B. What is the length of $\overline{AB}$?

8. Inside a circle, chords AB and CD intersect at P in right angles. Given that $AP = 6$, $BP = 12$ and $CD = 15$, find the radius of the circle.

9. Steven makes nonstandard checkerboards that have 29 squares on each side. The checkerboards have a black square in every corner and alternate red and black squares along every row and column. How many black squares are there on such a checkerboard?

10. John is organizing a race around a circular track and wants to put 3 water stations at 9 possible spots around the track. He doesn't want any 2 water stations to be next to each other because that would be inefficient. How many ways are possible?

11. In square $ABCD$, point E is chosen such that CDE is an equilateral triangle. Extend CE and DE to F and G on AB. Find the ratio of the area of $\triangle EFG$ to the area of $\triangle CDE$.

12. Let S be the number of integers from 2 to 8462 (inclusive) which does not contain the digit 1,3,5,7,9. What is S?

13. Let x, y be non zero solutions to $x^2 + xy + y^2 = 0$. Find

$$\frac{x^{2016} + (xy)^{1008} + y^{2016}}{(x+y)^{2016}}$$

14. A chess contest is held among 10 players in a single round (each of two players will have a match). The winner of each game earns 2 points while loser earns none, and each of the two players will get 1 point for a draw. After the contest, none of the 10 players gets the same score, and the player of the second place gets a score that equals to 4/5 of the sum of the last 5 players. What is the score of the second-place player?

15. Consider the sequence of positive integers generated by the following formula

$$\begin{aligned} a_1 &= 3 \\ a_{n+1} &= a_n + a_n^2 \text{ for } n = 2, 3, \ldots \end{aligned}$$

What is the tens digit of a_{1007}?

16. Let (x, y, z) be integer solutions to the following system of equations

$$\begin{aligned} x^2z + y^2z + 4xy &= 48 \\ x^2 + y^2 + xyz &= 24 \end{aligned}$$

Find $\sum x + y + z$ where the sum runs over all possible (x, y, z).

17. Given that $x + y = a$ and $xy = b$ and $1 \leq a, b \leq 50$, what is the sum of all a such that $x^4 + y^4 - 2x^2y^2$ is a prime squared?

18. In $\triangle ABC$, M is the midpoint of $\overline{AB}$, point N is on side $\overline{BC}$. Line segments $\overline{AN}$ and $\overline{CM}$ intersect at O. If $AO = 12$, $CO = 6$, and $ON = 4$, what is the length of OM?

19. Consider the following linear system of equations.

$$\begin{aligned} 1 + a + b + c + d &= 1 \\ 16 + 8a + 4b + 2c + d &= 2 \\ 81 + 27a + 9b + 3c + d &= 3 \\ 256 + 64a + 16b + 4c + d &= 4 \end{aligned}$$

Find $a - b + c - d$.

20. Consider flipping a fair coin 8 times. How many sequences of coin flips are there such that the string HHH never occurs?