Time limit: 60 minutes.
Instructions: This test contains 20 short answer questions.
No calculators.

1. Nikhil computes the sum of the first 10 positive integers, starting from 1. He then divides that sum by 5. What remainder does he get?

2. In class, starting at 8:00, Ava claps her hands once every 4 minutes, while Ella claps her hands once every 6 minutes. What is the smallest number of minutes after 8:00 such that both Ava and Ella clap their hands at the same time?

3. A triangle has side lengths 3, 4, and 5. If all of the side lengths of the triangle are doubled, how many times larger is the area?

4. There are 50 students in a room. Every student is wearing either 0, 1, or 2 shoes. An even number of the students are wearing exactly 1 shoe. Of the remaining students, exactly half of them have 2 shoes and half of them have 0 shoes. How many shoes are worn in total by the 50 students?

5. What is the value of $-2 + 4 - 6 + 8 - \cdots + 8088$?

6. Suppose Lauren has 2 cats and 2 dogs. If she chooses 2 of the 4 pets uniformly at random, what is the probability that the 2 chosen pets are either both cats or both dogs?

7. Let triangle $\triangle ABC$ be equilateral with side length 6. Points E and F lie on $\overline{BC}$ such that E is closer to B than it is to C and F is closer to C than it is to B. If $BE = EF = FC$, what is the area of triangle $\triangle AFE$?

8. The two equations $x^2 + ax - 4 = 0$ and $x^2 - 4x + a = 0$ share exactly one common solution for x. Compute the value of a.

9. At Shreymart, Shreyas sells apples at a price c. A customer who buys n apples pays nc dollars, rounded to the nearest integer, where we always round up if the cost ends in .5. For example, if the cost of the apples is 4.2 dollars, a customer pays 4 dollars. Similarly, if the cost of the apples is 4.5 dollars, a customer pays 5 dollars. If Justin buys 7 apples for 3 dollars and 4 apples for 1 dollar, how many dollars should he pay for 20 apples?

10. In triangle $\triangle ABC$, the angle trisector of $\angle BAC$ closer to $\overline{AC}$ than $\overline{AB}$ intersects $\overline{BC}$ at D. Given that triangle $\triangle ABD$ is equilateral with area 1, compute the area of triangle $\triangle ABC$.

11. Wanda lists out all the primes less than 100 for which the last digit of that prime equals the last digit of that prime's square. For instance, 71 is in Wanda's list because its square, 5041, also has 1 as its last digit. What is the product of the last digits of all the primes in Wanda's list?

12. How many ways are there to arrange the letters of `SUSBUS` such that `SUS` appears as a contiguous substring? For example, `SUSBUS` and `USSUSB` are both valid arrangements, but `SUBSSU` is not.

13. Suppose that x and y are integers such that $x \geq 5$, $y \geq 3$, and $\sqrt{x-5} + \sqrt{y-3} = \sqrt{x+y}$. Compute the maximum possible value of xy.

14. What is the largest integer k divisible by 14 such that $x^2 - 100x + k = 0$ has two distinct integer roots?

15. What is the sum of the first 16 positive integers whose digits consist of only 0s and 1s?

16. Jonathan and Ajit are flipping two unfair coins. Jonathan's coin lands on heads with probability $\frac{1}{20}$ while Ajit's coin lands on heads with probability $\frac{1}{22}$. Each year, they flip their coins at the same time, independently of their previous flips. Compute the probability that Jonathan's coin lands on heads strictly before Ajit's coin does.

17. A point is chosen uniformly at random in square $ABCD$. What is the probability that it is closer to one of the 4 sides than to one of the 2 diagonals?

18. Two integers are coprime if they share no common positive factors other than 1. For example, 3 and 5 are coprime because their only common factor is 1. Compute the sum of all positive integers that are coprime to 198 and less than 198.

19. Sumith lists out the positive integer factors of 12 in a line, writing them out in increasing order as $1, 2, 3, 4, 6, 12$. Luke, being the mischievious person he is, writes down a permutation of those factors and lists it right under Sumith's as $a_1, a_2, a_3, a_4, a_5, a_6$. Luke then calculates

$$\gcd(a_1, 2a_2, 3a_3, 4a_4, 6a_5, 12a_6).$$

Given that Luke's result is greater than 1, how many possible permutations could he have written?

20. Tetrahedron $ABCD$ is drawn such that $DA = DB = DC = 2$, $\angle ADB = \angle ADC = 90°$, and $\angle BDC = 120°$. Compute the radius of the sphere that passes through A, B, C, and D.