

Time limit: 15 minutes.

Instructions: This tiebreaker contains 3 short answer questions. All answers are positive integers. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

No calculators.

1. A particle starts moving from $(20, 0)$ along the line $x = 20$ at a speed of 1 unit per second in the positive y direction. Let $f(t)$ be the distance of the particle from the origin at time t - therefore, $f(20) = 20\sqrt{2}$. Then $f'(20)$ can be written in the form $\frac{p\sqrt{q}}{r}$, where p , q , and r are positive integers such that p and r are relatively prime and that q is square-free. Compute $p + q + r$.

2. For all real numbers x , let $f(x) = |x^2 + x|$. Let $I_1 = \int_{-2020}^0 f(x) \, dx$, and let $I_2 = \int_0^{2019} f(x) \, dx$. Then $|I_1 - I_2|$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

3. The integral

$$\int_3^4 \arcsin\left(\frac{\sqrt{x}}{2}\right) \, dx$$

can be written in the form $\frac{m\pi}{n} - \frac{p\sqrt{q}}{r}$, where m and n are relatively prime positive integers, p and r are relatively prime positive integers, and q is a square-free positive integer. Compute $m + n + p + q + r$.
