1. Let $f(x) = x^2 + x + 1$. Compute $f''(1)f'(1)f(1)$.

   Answer: 18

   Solution: We have that $f'(x) = 2x + 1$ and $f''(x) = 2$, so $f''(1)f'(1)f(1) = 2 \cdot 3 \cdot 3 = \boxed{18}$.

2. Let a be a positive integer. Compute

$$\int \left(\tan^{a+3}(x) + \tan^{a+2}(x) + \tan^{a+1}(x) + \tan^{a}(x)\right) \mathrm{d}x$$

   in terms of a. You do not need to include the $+C$ in your answer.

   Answer: $\frac{\tan^{a+2}(x)}{a+2} + \frac{\tan^{a+1}(x)}{a+1}$

   Solution: Let the integral be I. Reformat the integral.

$$\begin{aligned} I &= \int \left(\tan^{a+3}(x) + \tan^{a+1}(x) + \tan^{a+2}(x) + \tan^{a}(x)\right) \mathrm{d}x \\ &= \int \left((\tan^2(x) + 1)\tan^{a+1}(x) + (\tan^2(x) + 1)\tan^{a}(x)\right) \mathrm{d}x \\ &= \int \left((\sec^2(x))\tan^{a+1}(x) + (\sec^2(x))\tan^{a}(x)\right) \mathrm{d}x \\ &= \int \sec^2(x)\left(\tan^{a+1}(x) + \tan^{a}(x)\right) \mathrm{d}x\,. \end{aligned}$$

   Let $u = \tan(x)$ and compute the integral.

$$\begin{aligned} I &= \int (u^{a+1} + u^{a})\,\mathrm{d}u \\ &= \frac{u^{a+2}}{a+2} + \frac{u^{a+1}}{a+1}. \end{aligned}$$

   Substituting u back in, our answer is

$$I = \boxed{\frac{\tan^{a+2}(x)}{a+2} + \frac{\tan^{a+1}(x)}{a+1}}.$$

3. Compute the infinite sum

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\binom{n+1}{2}} = \frac{1}{\binom{2}{2}} - \frac{1}{\binom{3}{2}} + \frac{1}{\binom{4}{2}} - \frac{1}{\binom{5}{2}} + \cdots\,.$$

   Answer: $\mathbf{4\ln 2 - 2}$

Solution: We can rewrite the sum as

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\binom{n+1}{2}} &= \sum_{n=2}^{\infty} \frac{2(-1)^n}{n(n-1)} \\
&= 2\left(\sum_{n=1}^{\infty} \frac{1}{2n(2n-1)} - \sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n)}\right) \\
&= 2\left(\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n}\right) - \sum_{n=1}^{\infty} \left(\frac{1}{2n} - \frac{1}{2n+1}\right)\right) \\
&= 2\left(\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} + \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n}\right) \\
&= 2\left(2\sum_{n=1}^{\infty} \left(\frac{(-1)^{n-1}}{n}\right) - 1\right).
\end{aligned}$$

Using the Taylor expansion of $\ln(1+x)$ (or knowing the alternating harmonic series), this evaluates to $\boxed{4\ln 2 - 2}$.