

**Time limit:** 15 minutes.

**Instructions:** This tiebreaker contains 3 short answer questions. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

**No calculators.**

1. Let  $ABCDEFGH$  be a unit cube such that  $ABCD$  is one face of the cube and  $\overline{AE}$ ,  $\overline{BF}$ ,  $\overline{CG}$ , and  $\overline{DH}$  are all edges of the cube. Points  $I$ ,  $J$ ,  $K$ , and  $L$  are the respective midpoints of  $\overline{AF}$ ,  $\overline{BG}$ ,  $\overline{CH}$ , and  $\overline{DE}$ . The inscribed circle of  $IJKL$  is the largest cross-section of some sphere. Compute the volume of this sphere.
2. Let  $ABCD$  be a unit square. Points  $E$  and  $F$  are chosen on line segments  $\overline{BC}$  and  $\overline{CD}$ , respectively, such that the area of  $ABEFD$  is three times the area of triangle  $\triangle ECF$ . Compute the maximum possible area of triangle  $\triangle AEF$ .
3. In triangle  $\triangle ABC$ ,  $M$  is the midpoint of  $\overline{AB}$  and  $N$  is the midpoint of  $\overline{AC}$ . Let  $P$  be the midpoint of  $\overline{BN}$  and let  $Q$  be the midpoint of  $\overline{CM}$ . If  $AM = 6$ ,  $BC = 8$  and  $BN = 7$ , compute the perimeter of triangle  $\triangle NPQ$ .